GIOVE-A and GPS-35/36 orbit determination and analysis of dynamical properties based on SLR-only tracking data

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Abstract

SLR tracking data provided by the ILRS (International Laser Ranging Service) network are used to compute orbits of radio-navigation satellites equipped with laser retroreflectors : GPS-35 and GPS-36 for the American GPS constellation, and the first European GIOVE-A (Galileo In-Orbit Validation Element) satellite, launched in December 2005. The equations of motion are computed through an exhaustive dynamical model and is propagated with the two orbit determination softwares of the French GRGS (Groupe de Recherche de Géodésie Spatiale) group: GINS (for high frequency analyses), and CODIOR (for secular orbital elements analyses).

For each of these satellites, a set of SLR (Satellite Laser Ranging) data is processed and the results of the post-fit residuals analysis are shown. The orbit validation for GIOVE-A is based on overlaps between 2-day, 10-day and 30-day arcs calculated with the GINS software. The resulting 3D rms and radial residuals are the primary criteria for the internal accuracy of SLR orbits and may indicate possible dynamical perturbations such as orbit or attitude control manoeuvres. For GPS-35/36 satellites we compare two 10-day arcs to the precise IGS (International Global Navigation Satellite Systems Service) sp3 microwave final orbits. An offset of 2-3 cm in the radial direction appears between the two solutions and may reflect the effect of the non-homogeneity of the SLR tracking network. "Mean observed elements" are also provided.

Keywords: *GNSS, GIOVE-A, Satellite Laser Ranging, Solar radiation pressure modeling, mean orbital elements*

1. Introduction

GIOVE-A is the first satellite of the future GALILEO global navigation system. It has been developed by Surrey Satellite Technology Ltd and the ESA (*European Space Agency*) . It was launched from Baikonur Cosmodrome on 28 December 2005 and placed into a MEO (*Medium Earth Orbit*) with a semi major axis of 29600 km, an inclination of 56° and an eccentricity of 0.002**.** GIOVE-A is equipped with a LRR (*Laser Retro Reflector*) array having 76 corner cubes with a diameter of 27 mm each (*ESA-EUING-TN/10206*), which provides 40 % more return energy than GPS-35/36 LRR arrays (ILRS). The final constellation of Galileo will consist of 27 operational spacecrafts equipped with such identical LRR arrays. After the launch of GIOVE-A, ESA has requested ILRS an SLR campaign support during spring and summer 2006 (http://www.esa.int). The purpose of these campaigns is to provide data for the characterization of the satellite's on-board clock

The first of theses campaigns has taken place between 22 May and 24 July 2006, with the participation of 13 globally distributed SLR stations. This paper presents the results of the GIOVE-A orbit determination for this period. The orbit validation is based on overlaps of fitted SLR-only orbits of 2-day, 10-day and 30-day duration arcs.

The ILRS community is also actively tracking the only two GPS (*Global Positioning System*) satellites which have LRR arrays on-board, designated GPS-35 and GPS-36. The GPS satellites are equipped with LRR arrays of 32 corner cubes arranged in a flat panel of 19x29 cm (*Degnan and Pavlis, 1994; ILRS, 2004; Urschl et al., 2005*)**.** The altitude of GPS 35 and 36 is that of 20,195 km and 20,030 km respectively, with a 0.000 and 0.006 eccentricity and a 54 ° inclination for both.

In this study we are using 10 days of SLR data, for the two GPS satellites, in the period of 6th till 16th of June 2006. In this period most of the SLR stations where pointing to the GIOVE-A satellite and the SLR tracking data for the two GPS satellites have always been sparse. In this investigation the challenge consists in discovering the achievable orbit accuracy with sparse tracking data for the two GPS satellites. The analysis of SLR orbits of both GPS satellites is based on overlaps wrt the precise IGSsp3 orbits and the examination of difference residuals in the radial, normal and along-track direction. Transformation parameters between the fitted SLR arcs and the IGSsp3 orbits are adjusted.

Moreover, a propagation of the mean equations of motion, accounting for only the long periodic effects acting on the GIOVE-A orbit, has been led. This study provides the values of the mean observed elements, giving a mean value of each orbital parameter, and of the angles in particular (ascending node, argument of perigee, mean anomaly) for the 10-day arc.

The paper is organized such as follows. The analysis of the SLR-orbit estimation strategy and the solar radiation pressure modeling is outlined in Section 2. Section 3 describes the data set being used for GIOVE-A and GPS-35 and GPS-36 satellites. Section 4 analyses the results of the GIOVE-A internal orbit overlaps. Section 5 makes the analysis of the differences of the estimated SLR orbits of GPS-35/36 wrt IGSsp3 final microwave orbits for the period in question. Section 6 is dedicated to the analysis of GIOVE-A and GPS-35/36 orbit mean elements. Section 7 derives the necessary conclusions and summarizes the results.

2. SLR orbit estimation strategy

Our motivation to process the GIOVE-A and GPS-35/36 satellite SLR data on the period of June 2006 is two-fold: firstly we want to evaluate the implementation of the new box-and-wing SRP (Solar Radiation Pressure) model of GIOVE-A in our software GINS 6.1, and secondly to test the performances of SLR-only orbit determination for these 3 GNSS (Global Navigation Satellite System) satellites.

Our estimation strategy is based on a weighted least squares scheme. The present analysis is made by the orbit determination and analysis software package GINS 6.1 developed by the CNES (Centre National d’Etudes Spatiales) geodetic team of. In table (1) the ad-hoc models and estimated parameters are summarized.

The attitude model used for all three s/c is illustrated in Fig. 1. and corresponds to the following coordinate frame :

- The **Y-axis** points along the solar panels
- The **D-axis** points towards the sun
- The **X-axis** completes the system

For GIOVE-A and GPS-35/36 we have implemented a box and wing solar radiation pressure model including respectively 8 and 19 surfaces with a-priori reflectivity and specularity coefficients

GINS 6.1 soft. package	***GPS 35/36***	***GIOVE-A***
Datum definition	ITRF 2000, EOPC04	ITRF 2000, EOPC04
Tidal displacements	IERS03	IERS03
Gravity field	EIGEN_GL04S(20x20)	EIGEN_GL04S(20x20)
Atmospheric loading	ECMWF	ECMWF
Ocean loading	FES2004 (K2 cor.)	FES2004 (K2 cor.)
Troposphere	Marini-Murray	Marini-Murray
Solar Radiation Pressure	Box-and-wing	Box-and-wing
Albedo and infra-red	Analytical model (10°x10°)	Analytical model (10°x10°)
Satellite's retro-reflector offsets	x=-0.863, y=0.524, z=-0.658	x=0.828, y=0.655, z=-0.688
Attitude model	X, Y, D	X, Y, D
Numerical integration	Cowell 8th order, step size 180s	Cowell 8th order, step size180s
Parameter adjustment	6 orbital parameters, 1 SRP coeff., 1 Y-bias, 1 X, D per revolution (cos, sin)	6 orbital parameters, 1 SRP coeff., 1 Y-bias, 1 X, D per-revolution (cos, sin)

Table 1. *SLR-only orbit processing parameters for GPS-35/36 and GIOVE-A*

We have processed a set of 2-day, 10-day and 30-day arcs for the GIOVE-A satellite and two 10-day arcs for the GPS-35/36 satellites. Depending on the length of each arc, we include 1 per revolution terms for 2-day arcs (with constraints) and 5 per revolution terms (1 every 2d) for 10-day arcs in X, D directions. An additional acceleration along the s/c's Y-axis, the so-called Y-bias, is also adjusted.

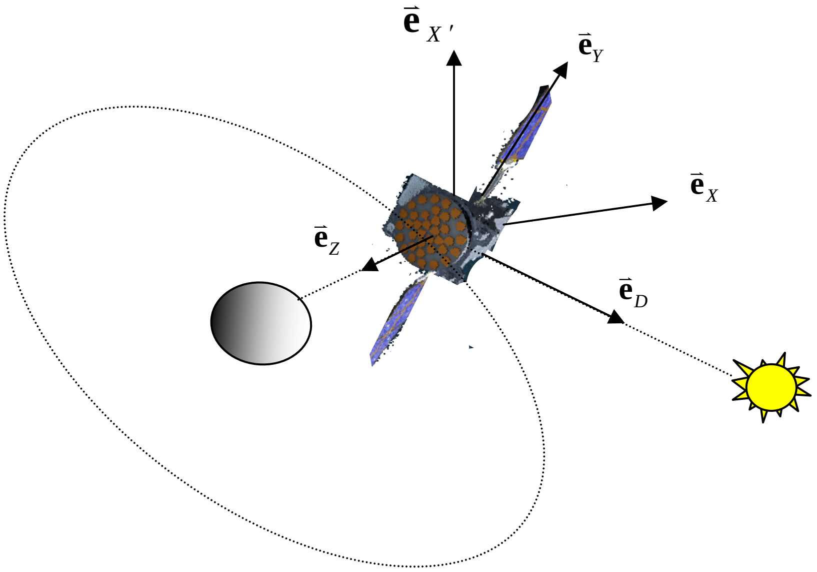


***Fig. 1**. The GIOVE-A and GPS-35/36 attitude model*

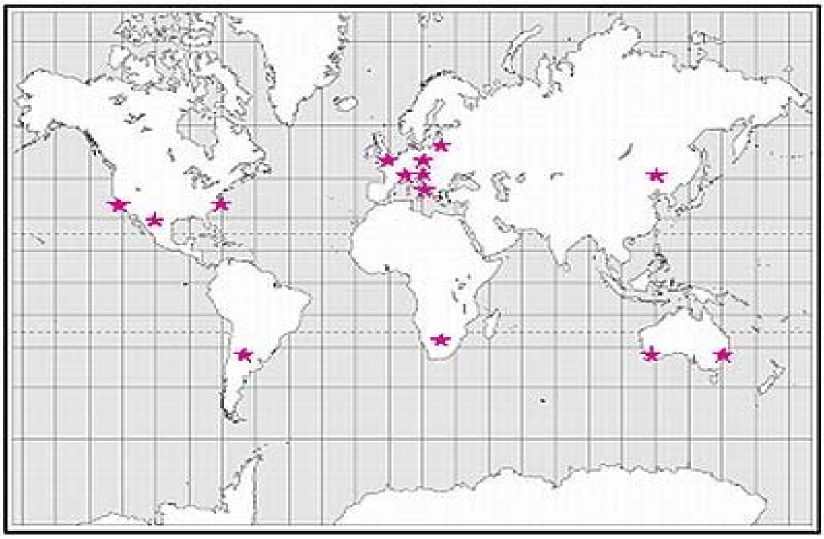


***Fig. 2.** The 13 SLR network stations distributed globally (ESA courtesy)*

3. Data set

Fourteen laser ranging stations (Fig. 2) participated in a campaign to track ESA's GIOVE-A satellite during spring and summer of 2006, providing invaluable data for the characterization of the satellite's on-board clock. The campaign was coordinated by ILRS and the GIOVE Processing Centre at ESA-ESTEC.

See www.esa.int/esaNA/SEM8QOKKKSE_index_2.html .

GIOVE-A satellite data from June to August 2006 used in this study have been processed. Figure 2 illustrates the distribution of the SLR tracking network. The total number of normal SLR points for this period arises up to 2311.

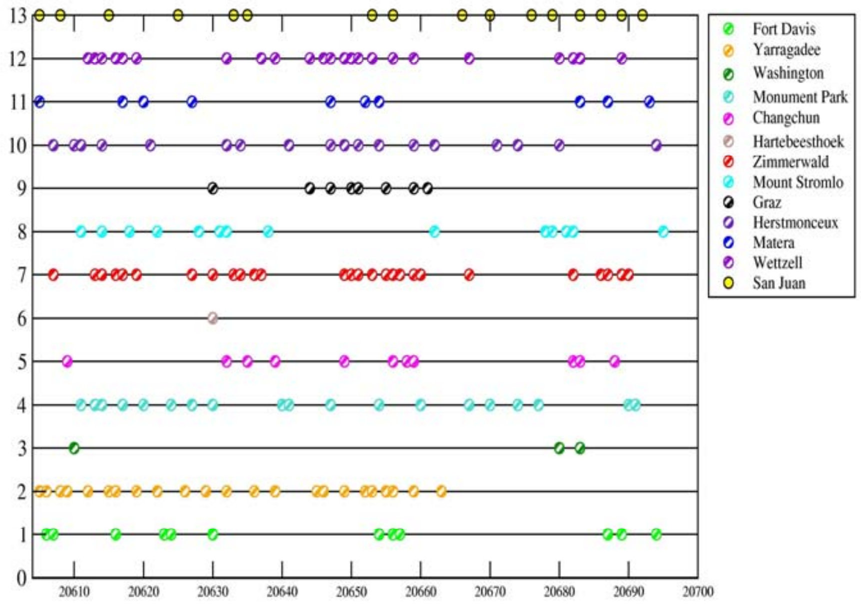


Fig. 3. *3 months (in Julian days 1950) of GIOVE-A SLR data from global tracking stations*

For GPS-35/36 we processed data from the period of June 2006 corresponding to a set of 306 and 402 normal points respectively. For the same period the amount of normal points for GIOVE-A is 900.

4. Orbit analysis of GIOVE-A

In this section we are examining:

- 1-day overlapping SLR-only sessions for GIOVE-A, from JULD50 (Julian day 1950) 20612 (2006/06/05) till JULD50 20623 (2006/06/19),
- a 10-day arc (2006/06/01.5-2006/06/11.5) over a 30-day arc (2006/06/01.5-2006/06/30.5)
- the overlaps with a 90-day arc expanding over the whole period of 3 months.

The illustration of the overlapping strategy is shown in Fig. 4.

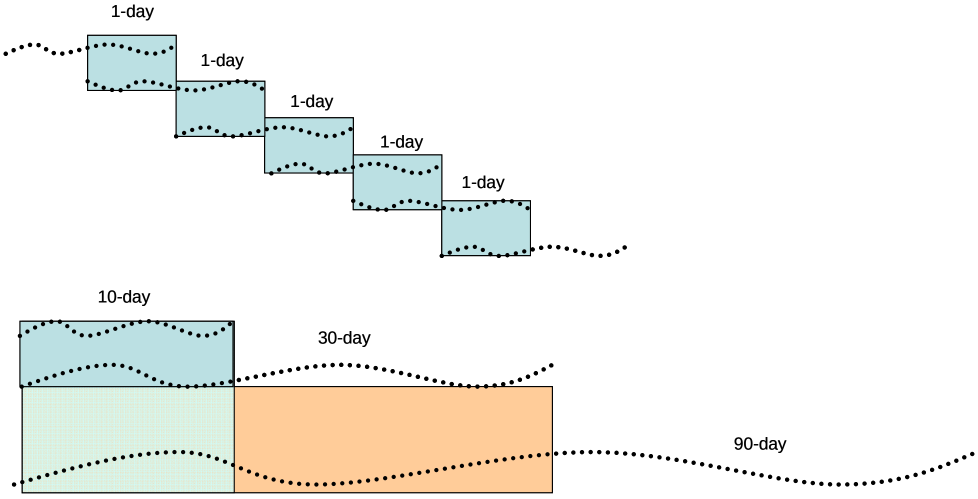


Fig. 4*. The overlapping periods of successive SLR arcs*

The evaluation criteria of the estimated orbit used are the root mean square misfit (RMS) (Eq. 1) and standard deviation (SD) of overlapping periods of successive arcs. An orbit overlap is defined by the comparison of the satellite's position vector between the common time-span of the two successive orbits (e.g. 1-day overlaps over 2 successive 2-day arcs).

$$rms_{misfit} = \sqrt{\frac{\left(\mathbf{x}^{arc1} - \mathbf{x}^{arc2}\right)}{n}} \qquad (1)$$

$$rms_{3D} = \sqrt{rms_{Radial}^{2} + rms_{Along}^{2} + rms_{Cross}^{2}}$$

Figure 5 shows the statistical results of the overlapping period of 2-day successive arcs.

For the arcs between JULD50 20611 (2006/06/08) and JULD50 20613 (2006/06/10), there is a significant change in the estimated accelerations, as well in the overlap mean difference and RMS. This implies that a dynamic perturbation like a manoeuvre occurred. In addition, a degradation of the mean difference of the SLR residuals appears at JJULD50 20620 (2006/06/16). This effect could be related to a reduction in the number of tracking stations for that epoch especially in the southern hemisphere.

The overlapping mean difference for the 2-day arcs is 43 cm in the Radial direction. Without accounting for the possible manoeuvre period it falls down to 14 cm. The same effect can be seen on the residual SD which decreases from 1.41 m to 32 cm for both 2 cases respectively.

Table 2 shows the orbit overlap misfit between a 10-day and a 30-day arc for the

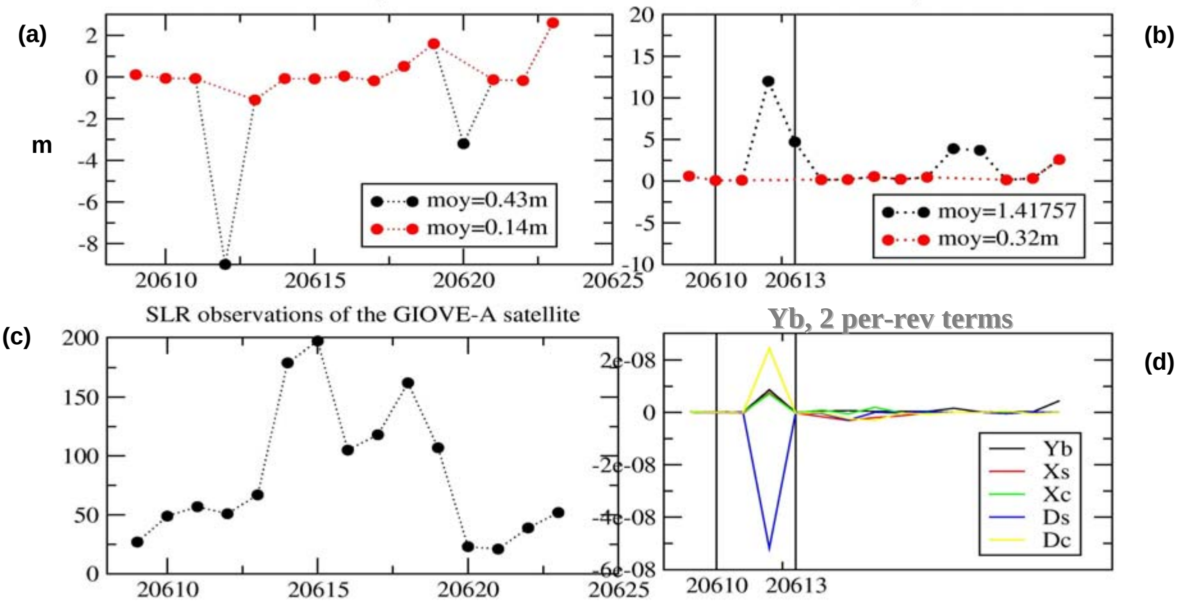


Fig. 5 *statistical results of the overlapping period of 2-day successive arcs: In (a) and (b) are illustrated the mean difference and the RMS misfit in the radial direction respectively. In black are the mean values (in m) including the perturbation days and in red are the mean values without the perturbation days. In (c) is the number of observations for every day and in (d) is the values of the empirical accelerations. Y-b is the Y bias, Xs and Xc are the sin and cos revolution terms in X direction, Dc and Ds are the sin and cos revolution terms in D direction. The perturbation has a stronger influence in the D direction revolution terms.*

period JULD50 20605 (2006/06/01) to 20615 (2006/06/11). The RMS of the satellite positions projected in the radial, normal and tangential directions are respectively 8cm, 45cm and 37cm.

The SLR residuals of a 10-day, 30-day and a 90-day arc are given figure 6 and lead to the same conclusions about the perturbations dates. All arcs agree in the residual level. Outliers up to 8m, verify the existence of dynamical perturbation event and appear in all arcs.

GIOVE-A RMS Misfits (cm)

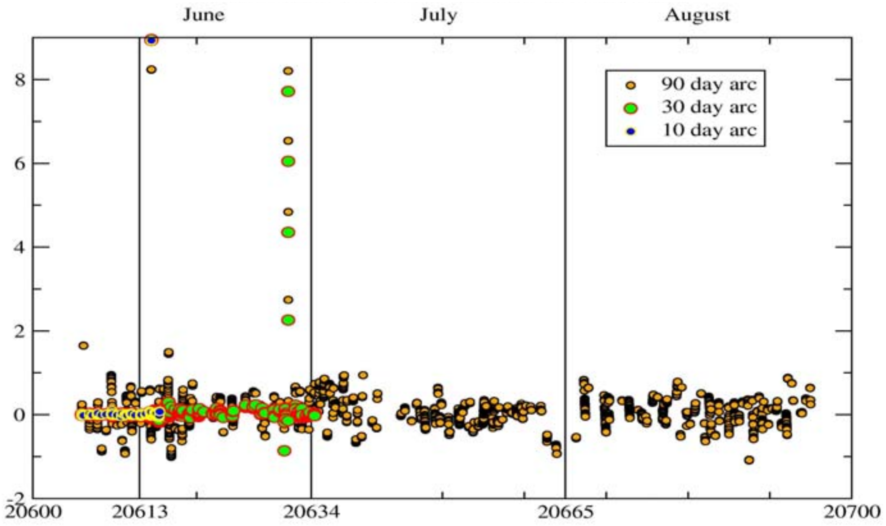
Earth Along (Tangential)	45.64
Earth Normal	37.46
Earth Radial	8.96

***Table 2**. GIOVE-A 10-day orbit overlaps from 2006/06/01.5 to 2006/06/11.5 over a 30-day arc from 2006/06/01.5 – 2006/06/30.5*

5. Orbit analysis of GPS 35/36

One 10-day SLR-only arc has been computed for GPS-35/ 36. The SLR data set spans from JULD50 20610 (2006/06/06) to 20620 (2006/06/16). As already mentioned, this period corresponds to a SLR campaign giving the priority to GIOVE-A tracking. This validation method has been very well known in the last 10 years and many studies, like Pavlis(1995), Appleby and Otsubo (2000), Hujsak et al. (1998) have investigated the undergoing problems of SLR sparse tracking orbit determination.

Tables 3(a) and 3(b) compare the adjusted orbits to the IGSsp3 final precise orbits in terms of position differences in the radial, normal and tangential directions. The RMS is at the level of 3 cm in radial, 47 cm in cross-track and 23 cm in along-track direction for GPS-35.



. ***Fig. 6:*** *SLR residuals for the 10-day, 30-day and 90-day arcs from the 1st of June*

GPS-35 RMS Misfits (cm)

Earth Along (Tangential)	23.81
Earth Normal	47.25
Earth Radial	3.24

Table 3 (a). *GPS-35 10-day SLR arc overlap wrt IGSsp3 final orbits*

GPS-36 RMS Misfits (cm)

Earth Along (Tangential)	9.55
Earth Normal	25.75
Earth Radial	2.03

Table 3 (b). *GPS-36 10-day SLR arc overlap wrt. IGSsp3 final orbits*

For the case of GPS-36 the level of agreement in comparison with the IGSsp3 radiometric orbits, is respectively in the radial, along-track, cross-track directions: 2-9-25 cm. Obviously, for GPS-35 and GPS-36, this result reflects the poor geographical distribution of SLR tracking stations. When one station in the southern hemisphere tracks GPS-36, for the same period, the factor of disagreement wrt IGSsp3 orbits drops down by a factor of 2.

Tx	-7.8 +/- 9.
Ty	-.4 +/- .9
Tz	**59.8 +/- 9.**
S (ppb)	.620124 x 10^{-9} +/- .375 x 10^{-9}
S (m)	**16.5 +/- 10**
Rx	-.3 +/- .1
Ry	.01 +/- .1
Rz	**-2.4 +/- .1**

Table 4 (a). *Helmert transformation wrt. the IGS microwave orbits for GPS-35 JJULD 20610-20620 in mm*

In order to further quantify any RF (Reference Frame) systematic differences, we applied a 7-parameter Helmert transformation between SLR-only orbits and IGSsp3 solutions. Table 4 (a) and 4 (b) summarize the statistics from this comparison.

Both translation coefficients in Z for GPS 35/36 are significative with 60 mm (± 10 mm) and 45 mm (± 5 mm) respectively. This offset may reflect systematic problems in either or both types of orbit as a result of non-homogeneity of SLR tracking stations in the global networks. In addition there is a factor of 8 in scale differences for GPS 35 and GPS 36 wrt the RF defined by IGSsp3 orbit. This statement is probably related to the poor number of southern tracking SLR tracking stations.

Tx	2.2 +/- 5.3
Ty	. 8 +/- 5.3
Tz	**45.3 +/- 5.3**
S (ppb)	.712820 x 10^{-10} +/- .2 x 10^{-9}
S (m)	**1.9 +/- 5.**
Rx	-.3 +/- .05
Ry	.04 +/- .05
Rz	**-1.4 +/- .05**

Table 4 (b). *Helmert transformation wrt. the IGS microwave orbits for GPS-36 JJULD 20610-20620 in mm*

Furthermore, the overall agreement of SLR-only orbits with sparse data wrt. the radiometric IGSsp3 final orbits, is 2 to 3 cm radially. The consistency of the RF arises up to 6-4 cm in translation along the z-axis.

6. Mean observed elements

A complementary study has been led to give the value of the mean elements of the orbits of GIOVE-A, GPS-35 and GPS-36, namely : the mean semi-major axis, the mean eccentricity and inclination for the metric variables (those providing the computation of secular effects induced on the angles), the mean ascending node, mean perigee and mean "mean anomaly". Such an approach leads up to an evaluation of the long term validity of gravitational and non gravitational models, and requires a data processing strategy where short periodic effects are removed from the osculating orbit, on each orbital element. This filtering approach has been carried out following the analytic part of the method, developed in (Exertier, 1990).The formulation of (Kaula, 1966) has been used to express the short period acting on the semi major axis, inclination, ascending node, and the one developed in (Deleflie, 2006) for the components of the eccentricity vector, because the investigated orbits are nearly circular.

Figures 7, 8, 9 show the temporal evolution of the mean metric elements of the GIOVE-A, GPS-35 and GPS-36 orbits, respectively. Table 5 gathers up some of these main elements, and Table 6 the main dynamic characteristics of these orbits which can be deduced from this study.

7. Conclusion and perspectives

The capability to estimate SLR-only orbits for GIOVE-A s/c has been implemented and evaluated in the GINS 6.1 CNES/GRGS software. The generated orbits are internally accurate to the level of 5-10 cm radially. This is the case when we are taking into account longer arc periods where orbit dynamics can absorb uniformly in the least square process a possible un-mapped perturbation such as s/c manoeuvres. Unknown manoeuvres are a critical issue for the s/c orbit determination.

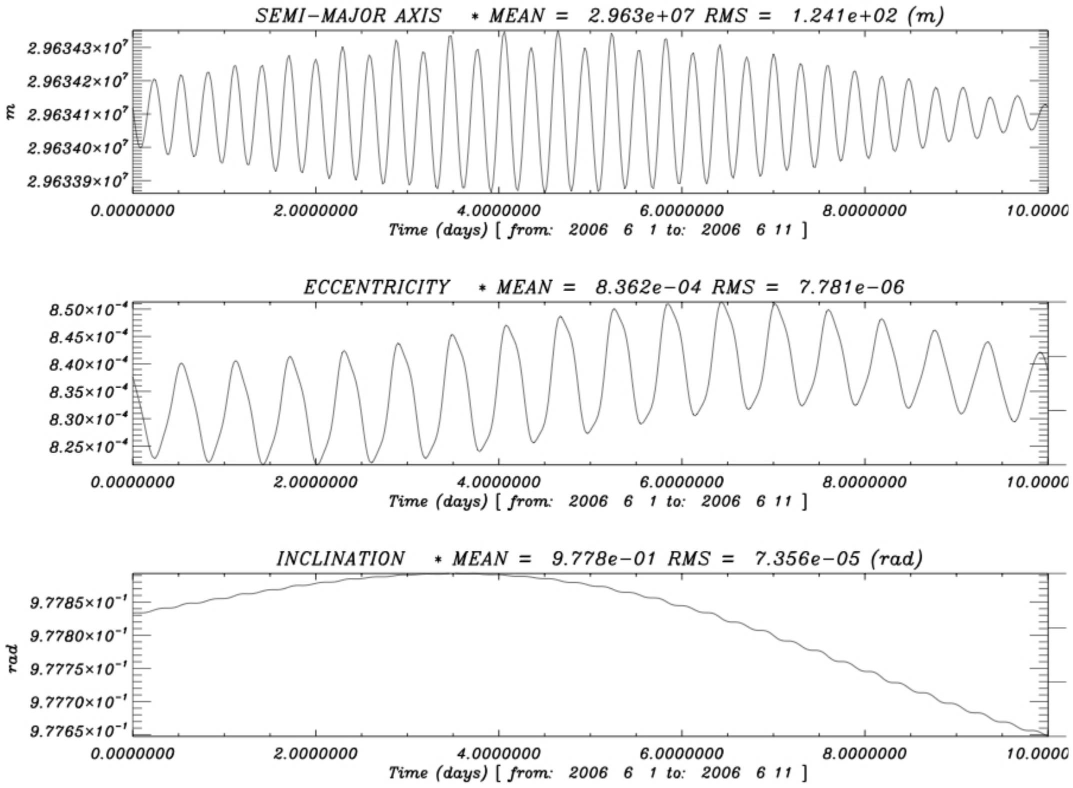


Fig 7. *Temporal evolution of the mean metric elements of the GIOVE-A orbit, from 2006, 1st of June to 2006, 11th of June*

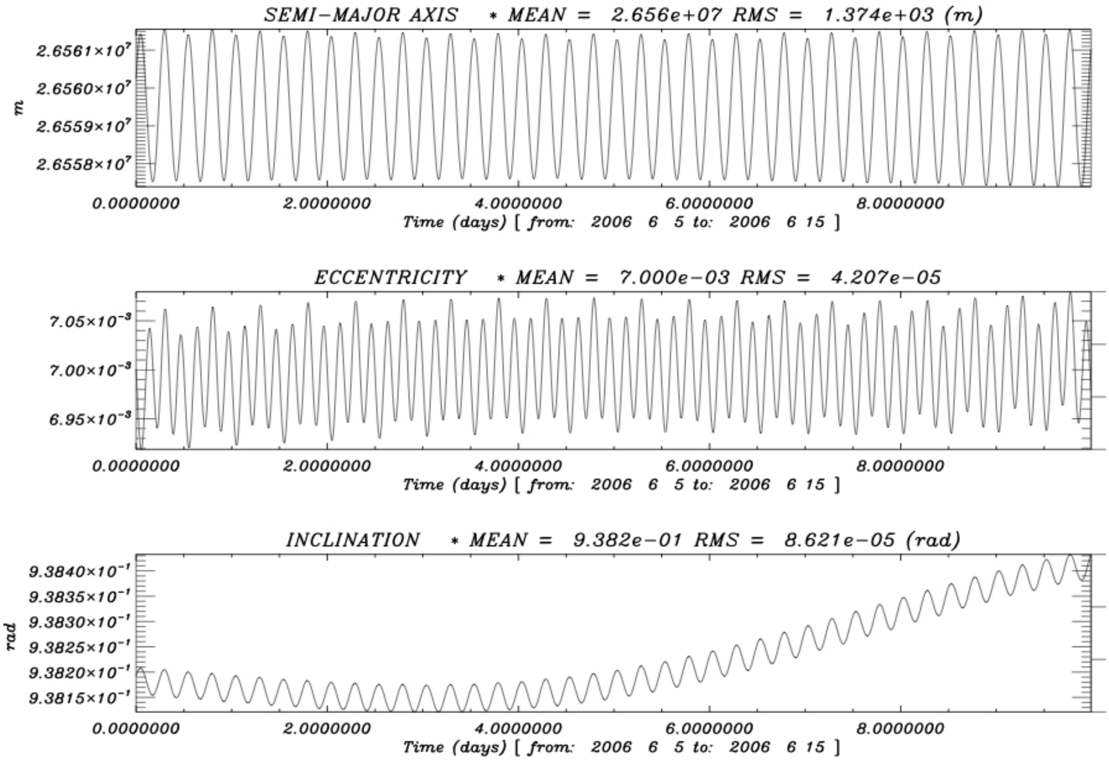


Fig 8. *Temporal evolution of the mean metric elements of the GPS-35 orbit, from 2006, 6th of June to 2006, 15th of June*

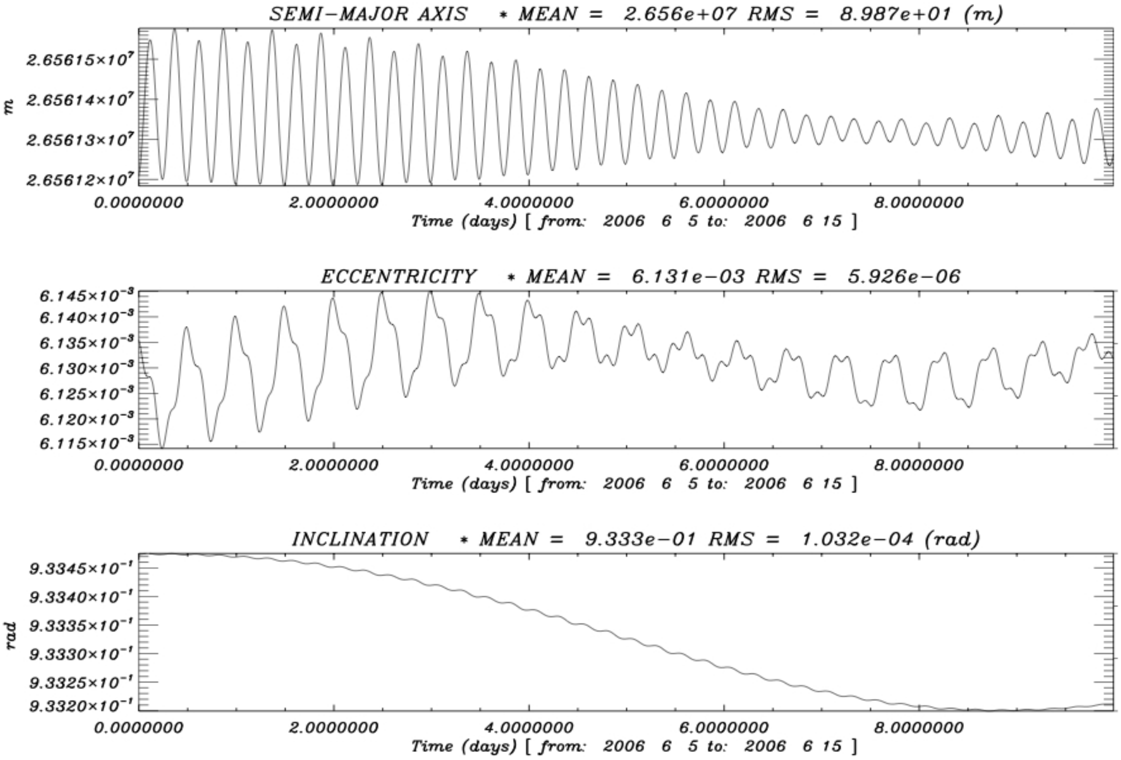


***Fig 9**. Temporal evolution of the mean metric elements of the GPS-36 orbit, from 2006, 6th of June to 2006 15th of June*

By comparing the results for the 90-day, 30-day, 10-day and 2-day orbits we believe that 2-day orbits are the most appropriate for further orbit dynamics investigation. Another critical aspect in the orbit determination of GIOVE-A s/c is the solar radiation pressure model (SRP). We are using an analytical box-and-wing SRP model with approximate specularity and reflectivity coefficients.

	Epoch. (Julian Days 1950)	Semimajor Axis *(m)*	Eccentricity	Inclination °	Ascending node *(rad)*	Argument of perigee *(rad)*	Mean anomaly *(rad)*
GIOVE-A	20605,5	0.29634118E+08	0.83763674E-03	56.025730°	0.32550034E+01	0.57163824E+01	0.11332092E+01
	20615,5	0.29634120E+08	0.83869966E-03	56.015079°	0.32504105E+01	0.57263379E+01	0.12404609E+01
GPS-35	20609,5	0.26560245E+08	0.70009131E-02	53.754485°	0.24052494E+01	0.10521913E+01	0.19530670E+01
	20619,5	0.26561274E+08	0.69619513E-02	53.768426°	0.23981146E+01	0.10572299E+01	0.23138993E+01
GPS-36	20609,5	0.26561208E+08	0.61354695E-02	53.484095°	0.35019663E+01	0.44620688E+01	0.31748379E+01
	20619,5	0.26561276E+08	0.61312841E-02	53.469107°	0.34948338E+01	0.44640863E+01	0.35254203E+01

***Table 5.** Mean observed elements for three orbits, deduced from an analytical filtering of the short periodic terms inside the osculating orbit adjusted on SLR-data.*

	Secular effectsd induced on			riod of revolution of			Altitude of	
	Asc. Node *(rad/s)*	Perigee *(rad/s)*	Mean anomaly *(rad/s)*	Asc. Node *(day)*	Perigee *(day)*	Mean anomaly *(min)*	Perigee *(km)*	Apogee *(km)*
GIOVE-A	-0.520220E-08	0.261182E-08	0.123762E-03	13979	27843	846	23231	23280
GPS-35	-0.807674E-08	0.510770E-08	0.145861E-03	9003	14238	718	19995	20367
GPS-36	-0.812694E-08	0.525981E-08	0.145852E-03	8948	13826	718	20020	20345

***Table 6.** Main characteristics of motion.*

A further improvement would be the adjustment of these coefficients in at least one year period time by making use, as well, of the most accurate radiometric observations in L1 and E5. Though an empirical model like those used by CODE orbit analysis center and implemented in the Bernese GPS software, would be further investigated

For GPS 35/36 the presented comparison to the IGSsp3 final orbits for the two 10-day arcs shows a high quality of SLR-only orbits derived with sparse data. RMS residuals are of the order of 2-3 cm radially, 5-10 cm in along and 25-40 cm in cross-track. The systematic patterns of the translation and scale parameters of the RF demonstrate the dependencies in the geographic distribution of the SLR network.

Finally, only two s/c of the GPS constellation are equipped with LRR arrays for orbit validation and the end of their life time could be within the next year. Nevertheless Europe’s satellite navigation system Galileo will offer this valuable opportunity of independent orbit validation procedures since all s/c of the constellation will be equipped with LRR arrays.

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